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TECHNICAL REPORT RD-RE-89-7

CALCULATION OF QUANTILES FOR HYPER-GAMMA, GENERALIZED GUMBEL, AND LOGNORMAL DISTRIBUTIONS

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Numerical FORTRAN algorithms are presented for αth quantiles of the distribution of an i.i.d. random variable distributed according to a hyper-Gamma, or a generalized Gumbel, or a lognormal probability density function. They are based on highly efficient routines for the evaluation of the incomplete Gamma and error functions. A program diskette will be made available upon request.

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TABLE OF CONTENTS

LIST OF TABLES

SUMMARY

Numerical FORTRAN algorithms are presented for αth quantiles of the distribution of an i.i.d. random variable distributed according to a hyper-Gamma, or a generalized Gumbel, or a lognormal probability density function. They are based on highly efficient routines for the evaluation of the incomplete Gamma and error functions. A program diskette will be made available upon request.

I. INTRODUCTION

To obtain information about a distribution it is frequently desirable to know its quantiles for certain levels α. In general terms, let X be an i.i.d. random variable with continuous probability density function (pdf) $f(x;P)$ where P denotes a parameter vector and $-\infty < x < \infty$.

Let

$$F(x) = \Pr\left(X \leq x\right) = \int_{-\infty}^{x} f(t;P)\, dt$$

be the associated cumulative distribution function (cdf). The αth quantile (non-exceedance probability at level α) of the distribution of X is that value x_α for which

$$F(x_\alpha) = \int_{-\infty}^{x_\alpha} f(t;P)\, dt = \alpha . \qquad (*)$$

It is also referred to as the (100α)th percentile of the distribution of X. Since $0 < F(x) < 1$ and since $F(x)$ is strictly monotonically increasing it is evident that α must satisfy the inequality $0 < \alpha < 1$ and that equation $(*)$ has exactly one solution. Equation $(*)$ may also be expressed in the form $x_\alpha = G(\alpha)$ where G is the inverse of F.

Unfortunately, there are only a few distributions for which the cdf can be inverted in closed form. A particular one for which this is possible is the exponential distribution with pdf $f(x;\lambda,b) = b^{-1} \exp -\xi$, $\xi = (x-\lambda)b^{-1}$. In general, therefore, it is necessary to solve the quantile equation $(*)$ numerically for given levels α once the distribution pdf $f(x;P)$ has been specified.

This report presents efficient algorithms for the numerical solution of the quantile equation $(*)$ for three specific classes of probability distributions:

(i) <u>Hyper-Gamma</u>:

$$f(x;P) = \begin{cases} \dfrac{\beta}{b\,\Gamma(a)}\, \xi^{-p} \exp -\xi^{\beta} , \ \xi = (x-\lambda)b^{-1} , x > \lambda , \\[2ex] 0 , x < \lambda , \end{cases} \qquad (1.1)$$

$a = (1-p)\beta^{-1}$, $P = (\lambda, b, \beta, p)$, where λ is the real shift parameter, $b > 0$ the scale parameter, $\beta > 0$ the terminal shape parameter, and $p < 1$ the initial shape parameter.

(ii) Generalized Gumbel (Extreme Value Type I for Maximum Elements):

$$f(x;P) = \frac{\beta^{\beta}}{b\Gamma(\beta)} \exp - \beta\left(e^{-\xi} + \xi\right), \quad \xi = (x-\lambda)b^{-1}, \; -\infty < x < \infty, \tag{1.2}$$

$P = (\lambda, b, \beta)$, where λ is the real shift, $b > 0$ is scale, and $\beta > 0$ is shape. For $\beta = 1$ the classical Gumbel pdf results.

(iii) Lognormal:

$$f(x;P) = \begin{cases} \frac{1}{(x-\lambda)\sigma\sqrt{2\pi}} \exp - \left[\left(\log(x-\lambda) - \mu\right)^2 / 2\sigma^2\right], & x > \lambda, \\ 0, & x \leq \lambda. \end{cases} \tag{1.3}$$

$P = (\lambda, \sigma, \mu)$, λ is real shift, $\sigma > 0$ is shape, μ is real scale. (The scaling property of μ becomes apparent if one sets $\mu = \log b$, $b > 0$.)

For the three distribution classes defined by the pdf's (1.1), (1.2), and (1.3), parameter estimation algorithms for given data have recently been made available [1], [2,3], [4]. Their parameter vector outputs could directly be used as inputs for the quantile algorithms offered in this report in order to achieve a fully automated distribution assessment package if it were augmented by a goodness-of-fit evaluation algorithm. However, goodness-of-fit considerations have been beyond the scope of the investigations that resulted in the parameter estimation methods, and they are outside the scope of the present investigation. Furthermore, different analysts may have different objectives depending on the ultimate use of the numerical results.

Therefore, the three quantile algorithms are presented in stand-alone versions. To facilitate easy hook-up to the existing parameter estimation algorithms they have been packaged individually. A program diskette will be made available upon request.

II. THE QUANTILE EQUATIONS

Throughout this section let $y = x - \lambda$, and $\eta = yb^{-1}$.

(i) The cdf for the hyper-Gamma distribution class (1.1) takes the form

$$F(y) = \frac{1}{\Gamma(a)} \gamma\left(a, \eta^{\beta}\right), \quad \eta > 0,$$

with $a = (1-p)\beta^{-1}$, $\Gamma(x)$ the (complete) Gamma function, and $\gamma(\nu,x)$ the incomplete Gamma function with upper integration limit x.

With $\eta^{\beta} = u > 0$ the general quantile equation (*) reduces to $\gamma(a,u) - \alpha\Gamma(a) = 0$. For computational efficiency it is useful to transform this equation into the more convenient form

$$\varphi(u) = \gamma^{*}(a,u) - \alpha \exp - (a \log u) = 0, \quad \alpha \in (0,1), \tag{2.1}$$

where $\gamma^{*}(a,u)$ is the reduced incomplete Gamma function, $\gamma^{*}(a,u) = \gamma(a,u)/[u^{a}\Gamma(a)]$.

If u_{α} is the (unique) solution of (2.1), the corresponding x value is given by

$$x_{\alpha} = \lambda + b \exp\left(\beta^{-1} \log u_{\alpha}\right). \tag{2.1a}$$

(ii) The cdf for the generalized Gumbel distribution class (1.2) is

$$F(y) = \frac{1}{\Gamma(\beta)} \Gamma\left(\beta, \beta e^{-\eta}\right), \quad \eta \in \mathbb{R},$$

with $\beta > 0$, $\Gamma(\nu,x)$ the complementary incomplete Gamma function, $\Gamma(\nu,x) = \Gamma(\nu) - \gamma(\nu,x)$.

With $\beta \exp -\eta = u > 0$ equation (*) now takes the form $\Gamma(\beta,u) - \alpha\Gamma(\beta) = 0$ which can be changed into the more convenient form

$$\varphi(u) = \gamma^{*}(\beta,u) - (1-\alpha) \exp - (\beta \log u) = 0, \quad \alpha \in (0,1). \tag{2.2}$$

If u_{α} is the (unique) solution, one obtains the corresponding x value as

$$x_{\alpha} = \lambda + b \log(\beta u_{\alpha}^{-1}). \tag{2.2a}$$

(iii) Under the transformation $(\sigma\sqrt{2})^{-1}\log\eta = u \in \mathcal{R}$, ($b = \exp\mu$), the cdf for the lognormal distribution class (1.3) can be written as

$$F(y) = \frac{1}{2}(1 + \text{erf } u)$$

with

$$\text{erf } u = \frac{2}{\sqrt{\pi}} \int_0^u \exp - t^2 dt \ .$$

The quantile equation to be solved then is

$$\varphi(u) = \text{erf } u - (2\alpha - 1) = 0 \ , \ \alpha \in (0,1) \ . \tag{2.3}$$

If u_α is the (unique) solution, the corresponding x value is given by

$$x_\alpha = \lambda + \exp\left(\mu + \sigma u_\alpha \sqrt{2}\right) \ . \tag{2.3a}$$

III. DESCRIPTION OF THE ALGORITHMS

In each of the three cases discussed here the user has to provide as input a complete set of admissible distribution parameter values. The user then has the option either to set α at any desired level in the interval (0,1) or to ask for quantiles for a pre-established set of fairly standard α values. (The program printout should be consulted for details.)

(i) The solution algorithms for equation (2.1) is started by evaluation of $\varphi(1)$. If $\varphi(1) = 0$, then $u_0 = 1$, and $x_0 = \lambda + b$ according to (2.1a). If $\varphi(1) < 0$, the root u_0 of (2.1) is greater than unity. In this case the function $\varphi(u)$ is evaluated at the points of the sequence $u_\nu = 1 + 10^{-1} (1.618)^{\nu-1}$ (ν=1, 2, ...) until the root u_α is bracketed by change in sign of φ. If $\varphi(1) > 0$, the root $u_\alpha \in (0,1)$. Now the search sequence $u_\nu = 1 - 2^{\nu-1}/(2^{\nu-1} + 99)$ (ν=1, 2, ...) is used to achieve root bracketing.

Once u_α has been bracketed, Brent's method [5: Chap. 7.3] performs final calculation of u_α. The algorithm returns x_α by (2.1a).

The function $\gamma^*(a,u)$ is evaluated by means of a routine provided in [6: Chap. 45.8].

(ii) Because of the structural similarity between equations (2.1) and (2.2) the solution strategy employed relative to the first carries over directly to the other. The only difference is that now $\varphi(1) > 0$ implies that $u_\alpha > 1$, $\varphi(1) < 0$ implies that $u_\alpha \in (0,1)$.

(iii) The solution algorithm of equation (2.3) is initiated by evaluation of $\varphi(0)$. If $\alpha = 1/2$, then $u_{1/2} = 0$, and $x_{1/2} = \lambda + \exp \mu$ according to (2.3a). If $\varphi(0) < 0$, ($1/2 < \alpha < 1$), the root $u_\alpha > 0$. The search sequence $u_\nu = 10^{-1} (1.618)^{\nu-1}$ (ν=1,2, ...) is used for root bracketing. If $\varphi(0) > 0$ ($0 < \alpha < 1/2$), $u_\alpha < 0$. The search sequence $u_\nu = - 10^{-1} (1.618)^{\nu-1}$ (ν=1, 2, ...) is used now. Brent's method is employed again. Error function evaluation is done by a routine given in [6: Chap. 40.8].

<u>Warning</u>: Because of properties of the functions γ^* and erf it may happen that either of the three quantile algorithms is unable to produce a root of the corresponding equation $\varphi(u) = 0$. This is true if the parameters a and β in equations (2.1) and (2.2), respectively, are small or large. Relative to equation (2.3) failure may occur if α or $1-\alpha$ is small.

IV. EXAMPLES

Example 1.

Three-parameter hyper-Gamma estimation (with λ assumed to be zero) has been performed in [1; Sec. 7, Ex. 2, Table 2.1] for (simulated) observations of interarrival times (in minutes) at a drive-up banking facility over a 90-minute period [7; Chap. 5.31, Ex. 5.1, p. 184]. The smallest and largest observations are $x_{min} = 0.1$, $x_{max} = 1.96$. Quantiles for various α levels for the moment (M) and maximum-likelihood (ML) parameters are shown in Table 1 of Sec. V. The x_{α} values are in minutes.

Example 2.

Four-parameter hyper-Gamma estimation has been performed on another data set [1; Sec. 7, Ex. 3, Table 3.2] which represents observations of daily mean temperatures (in ^{0}F) at Shemya, Alaska, during the winters of 1960-1979, with $x_{min} = 18$, $x_{max} = 40$. The results for M and ML parameters are displayed in Table 2. The x_{α} values are in ^{0}F.

Example 3.

Three-parameter generalized Gumbel estimation results for observations of annual 24-hour maximum rainfalls (in points, $x_{min} = 1.54$, $x_{max} = 1105$) at Sidney, Australia, over the period 1859-1945, have been provided in [3; Sec. 7, Ex. 1, Table 2]. The original data are from [8]. Table 3 shows the M and ML quantiles. The x_{α} values are in points.

Example 4.

Another three-parameter generalized Gumbel estimation has been done in [3; Sec. 7, Ex. 2, Table 5] for observations of annual peak gust winds [in kts] at Argentia Naval Air Station, Newfoundland, over the period 1941-1963 (original data from [9; Chap. 4.12], $x_{min} = 62$, $x_{max} = 91$). Results are shown in Table 4; x_{α} is in kts.

Example 5.

Two-parameter lognormal estimation (λ assumed to be zero) has been performed in [4; Sec. VIII, Ex. 4, Table 4.3] on observations of rainfall totals (in inch, $x_{min} = 2.5$, $x_{max} = 18.5$) for sets of four consecutive months at Camden Square, London, over the period 1870-1943 [10; Chap. 7.5, Ex. 7.511]. Table 5 shows the results; x_{α} in inch.

Example 6.

The report [4; Sec. VIII, Ex. 5, Table 5.3] contains another two-parameter (λ=0) lognormal estimation for data given in [11; Ex. 10, p. 47]. They represent weekly precipitation sums observations (in 10^{-2} inch, x_{min} = 25, x_{max} = 825) at Kwajalein, Marshall Islands, during the summers 1949-1958. Quantile results are displayed in Table 6; x_{α} is in 10^{-2} inch.

TABLES

TABLE 1. M and ML Quantiles, Drive-Up Bank

DISTRIB.TYPE: Four Parameter Hyper-Gamma

	Mom. Estim.	ML. Estim.
LAMBDA =	0.000000	0.000000
BETA =	1.625230	0.967295
BVAL =	0.809469	0.354808
PVAL =	0.265411	-0.069105

Cum.Pct.Lvl.	Quantiles	
0.010%:	0.000002	0.000067
0.100%:	0.000057	0.000581
1.000%:	0.001300	0.005038
2.000%:	0.003339	0.009703
5.000%:	0.011628	0.023324
10.000%:	0.029920	0.046070
25.000%:	0.105559	0.119964
50.000%:	0.288488	0.280036
75.000%:	0.583231	0.550817
90.000%:	0.927675	0.908438
95.000%:	1.161985	1.179608
98.000%:	1.446185	1.539171
99.000%:	1.646047	1.812012
99.900%:	2.243029	2.723128
99.990%:	2.768327	3.639656

TABLE 2. M and ML Quantiles, Shemya

DISTRIB.TYPE: Four Parameter Hyper-Gamma

	Mom. Estim.	ML. Estim.
LAMBDA =	12.750255	13.517533
BETA =	7.575647	7.952291
BVAL =	22.152744	21.772694
PVAL =	-3.979800	-3.593978

Cum.Pct.Lvl.	Quantiles	
0.010%:	16.163086	16.377214
0.100%:	18.169350	18.238099
1.000%:	21.355516	21.310090
2.000%:	22.641807	22.579652
5.000%:	24.646488	24.583222
10.000%:	26.441695	26.396393
25.000%:	29.317097	29.317104
50.000%:	32.164208	32.196963
75.000%:	34.593508	34.620922
90.000%:	36.462796	36.458289
95.000%:	37.461554	37.429782
98.000%:	38.495249	38.428114
99.000%:	39.137810	39.045228
99.900%:	40.772239	40.603981
99.990%:	41.966407	41.733911

TABLE 3. M and ML Quantiles, Sidney

DISTRIB.TYPE: Three Parameter Genl.Gumbel

	Mom. Estim.	ML. Estim.
LAMBDA =	348.336000	335.783000
BETA =	1.056140	0.735367
BVAL =	143.341000	110.752000

Cum.Pct.Lvl.	Quantiles	
0.010%:	35.486434	66.041079
0.100%:	76.136252	100.207878
1.000%:	133.353174	149.118178
2.000%:	156.320799	169.072924
5.000%:	193.824197	202.093947
10.000%:	230.716691	235.141751
25.000%:	301.590770	300.316124
50.000%:	397.861599	392.554633
75.000%:	519.170057	514.573585
90.000%:	656.844906	659.252220
95.000%:	755.103127	765.194247
98.000%:	881.978195	903.887393
99.000%:	976.912277	1008.450385
99.900%:	1290.236179	1355.340981
99.990%:	1602.838134	1702.132745

TABLE 4. M and ML Quantiles, Argentia

DISTRIB.TYPE: Three Parameter Genl.Gumbel

	Mom. Estim.	ML. Estim.
LAMBDA =	74.468400	74.528300
BETA =	4.410460	4.757460
BVAL =	13.390300	13.941600

Cum.Pct.Lvl.	Quantiles	
0.010%:	56.646226	56.499598
0.100%:	59.210458	59.105693
1.000%:	62.609585	62.551503
2.000%:	63.907460	63.864472
5.000%:	65.947261	65.924875
10.000%:	67.860895	67.854326
25.000%:	71.294876	71.308055
50.000%:	75.504766	75.526940
75.000%:	80.199135	80.211754
90.000%:	84.907464	84.890403
95.000%:	87.973928	87.927217
98.000%:	91.672466	91.579797
99.000%:	94.295208	94.163588
99.900%:	102.384949	102.103430
99.990%:	109.954114	109.497965

TABLE 5. M and ML Quantiles, Camden Square

DISTRIB.TYPE: Two-Parameter Log-Normal

	Mom. Estim.	ML. Estim.
SIGMA =	0.321706	0.341545
MU =	2.085390	2.081320

Cum.Pct.Lvl.	Quantiles	
0.010%:	2.432599	2.250401
0.100%:	2.977976	2.789512
1.000%:	3.807565	3.621063
2.000%:	4.156555	3.974394
5.000%:	4.740918	4.570070
10.000%:	5.328699	5.173826
25.000%:	6.477940	6.365873
50.000%:	8.047729	8.015042
75.000%:	9.997923	10.091451
90.000%:	12.154176	12.416516
95.000%:	13.661058	14.056874
98.000%:	15.581641	16.163695
99.000%:	17.009807	17.740896
99.900%:	21.748315	23.029440
99.990%:	26.624183	28.546419

TABLE 6. M and ML Quantiles, Kwajalein

DISTRIB.TYPE: Two-Parameter Log-Normal

	Mom. Estim.	ML. Estim.
SIGMA =	0.677532	0.819763
MU =	5.211890	5.143320

Cum.Pct.Lvl.	Quantiles	
0.010%:	14.763196	8.122255
0.100%:	22.604720	13.599893
1.000%:	37.928958	25.438559
2.000%:	45.622882	31.808471
5.000%:	60.186557	44.475116
10.000%:	76.984015	59.904489
25.000%:	116.152224	98.533664
50.000%:	183.440433	171.283487
75.000%:	289.709411	297.746288
90.000%:	437.108827	489.746816
95.000%:	559.101469	659.650503
98.000%:	737.577083	922.333953
99.000%:	887.195286	1153.289884
99.900%:	1488.644532	2157.225330
99.990%:	2279.343398	3612.054976

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